EVALUATION OF OPERATIONAL SAFETY OF AVIONIC SERVO-ASSISTED STEERING SYSTEMS WITH USE OF THE NYQUIST STABILITY CRITERION

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Abstract: The paper outlines an alternative method of evaluation of operational safety of avionic servo-assisted steering systems as well as of identification of technical condition of such systems on the basis of the stability analysis of a hydraulic servomechanism (servo-amplifier). Application of the enhanced linear method, widely used in the theory of automatic regulation, was proposed in order to check stability of hydraulic servomechanism. The method is reduced to application of the simplified Laplacian operator function to determine transfer characteristics of the open-loop system. The equation for transmittance of the hydraulic system for servo-assisted steering of an aircraft serves as a background for analysis of stability by means of enhanced linear methods. On the basis on the operator function it was possible to draw up the Nyquist graphs (amplitude vs. phase characteristic curves) for the servomechanism in the poplar co-ordinate system (Nyquist hodographs). It was demonstrated that pressure in the hydraulic system for servo-assisted steering of an aircraft is the defining parameter for the proposed method. The paper includes results of stability analysis for the servo-assisted hydraulic system for longitudinal steering of the Su-22 aircraft.

1. Introduction

Exploitation of hydraulic systems for servo-assisted steering of aircrafts as well as subassemblies and modules of such systems is performed in complex circumstances, so there is a great number of factors that adversely affect technical characteristics of the systems and modules as time goes by.

Hydraulic systems for servo-assisted steering of aircrafts are exposed to variation of the input function and interferences to much higher degree than other servomechanisms. The variations are imposed by aerodynamic loads that affect steering planes of aircrafts and that change during the flight. Avionic safety requires stability of the hydraulic systems for servo-assisted steering, i.e. after every deviation, in spite of the source and magnitude of disturbances, the system should spontaneously restore the equilibrium state. Hence the values of hydraulic pressure in the system when the hydraulic servo valves operate in a steady manner are an essential factor that affect the assessment of technical condition of the servo-assisted steering systems for aircrafts. This requirement implies the need and the obligation to carry out stability analysis of servo valves incorporated into the hydraulic system for servo-assisted steering of aircrafts. Analysis of stability with due
consideration for all the phenomena that take place during the process of aircraft steering is difficult due to great number of factors that affect the course of the process. In addition, many factors that determine transient states subject to significant variation over the lifetime of the steering system. However, investigation of the hydraulic servomechanism can be accomplished by means of linear methods that are well-developed in the theory of automatic control and regulation [1, 2] i.e. using the simplified operator transfer function for an open system. Later on, the characteristic curves of the hydraulic servomechanism can be drawn on the background of the operator function in question. Frequency characteristics for the hydraulic servomechanism can be plotted within the polar coordinate system (the Nyquist hodograph) or the Black plane (the phase in degrees along the X-axis and the module in dB along the Y-axis).

The best approach to evaluation of technical condition of the hydraulic servomechanism for the servo-assisted steering system is the use of such a structural parameter of the system as pressure. That is why the most suitable method for stability analysis of the hydraulic servomechanism is the one that bases on the amplitude vs. phase characteristic that is already plotted in the polar (Nyquist) coordinate system. The choice of the amplitude vs. phase characteristic in the polar coordinate system is convenient due to the fact that the method involves pressure in the hydraulic system for servo-assisted steering as a major parameter of the analysis. The present paper discloses results of stability analysis of the hydro-mechanical servomechanism BU-250 from Se-22 aircraft.

The developed method that is described in this paper can be also applied to simulation tests of newly-designed hydraulic systems for servo-assisted steering of aircrafts.

2. The hydraulic servomechanism for servo-assisted steering of an aircraft

A hydraulic servomechanism is understood as every control unit with a closed feedback loop and hydraulic amplification of power. The hydraulic servomechanism is designed to transform an input signal of electric or mechanical nature into an output hydraulic signal whereas the output power is proportional to the input excitation.

The steering system of Su-22 aircraft may incorporate two types of hydro-mechanic servomechanisms, i.e. the hydraulic servomechanism RA-30 (see. Fig. 1) that uses an electric signal as the input function or the hydro-mechanical servomechanism BU-250 (see. Fig. 2) where a mechanical parameter is delivered to the input. The design and operation principle of the hydraulic servomechanism RA-30 is explained in [3].
The present paper shall deal with a hydro-mechanical servomechanism with a double-action symmetrical linear actuator and mechanical output signal and location-dependent amplification factor of the servomechanism where the amplification factor is equal to one. The design and operation principle of the hydraulic servomechanism BU-250 is explained.
The feedback that transfers information on location or velocity to the input of the servomechanism, i.e. the idea of automatic control in the hydro-mechanical servomechanism BU-250, is carried out by a rigid connection where the slider of a flow distributor is connected with a piston rod of the actuator by means of an inflexible lever. The schematic diagram of a hydraulic servomechanism with mechanical feedback is shown on Fig. 3. The hydro-mechanical servomechanism BU-250 acts as an actuating part of the irreversible control system for the all-moving tail plane of the Su-22 aircraft. It means that when the feeding pressure of the hydro-mechanical servomechanism decays, the aircraft becomes uncontrollable. Inoperability of the hydro-mechanical servomechanism, which is manifested by lockout of the slider in the distributor, implies impossibility of either longitudinal or transverse steering of the aircraft whilst excessive friction of the distributor slider is associated with tough movements of the control stick and “walking” of the control stick with very small deflections of stabilizer planes.

The hydro-mechanical servomechanism, as shown on Fig. 2, can be described by three equations: kinematical equation, balance of flow and balance of forces.

3. Transmittance of the hydraulic system for servo-assisted aircraft steering

As it results from the operational analysis of a hydraulic servomechanism for the aircraft control system, the movements of the servomechanism cylinder are controlled by the distribution ratio of the flow control servo-valve. The structural diagram of the servomechanism for the servo-assisted aircraft control system is shown on Fig. 4. The following basic symbols are used to denote operational parameters of the mechanism:
\( x(t) \) – input parameter (travel of the slider in the distributing valve of the hydraulic servomechanism),
\( y(t) \) – output parameter (travel of the cylinder piston in the hydraulic servomechanism),
\( e \) – distribution ratio of the flow control servo-valve (\( e = x - y \)),
\( H_1 \) – transmittance of the hydraulic system for servo-assisted longitudinal steering of the aircraft.

Fig. 4. The equivalent structural diagram of a servomechanism for the aircraft control servomechanism with hydraulic servo-assistance.

The kinematical feedback is provided by mechanical connection of the strand that actuates the distributor slider with the movable part of the cylinder in the hydraulic servomechanism. When the position of the actuating strand is denoted by \( z \) while the position of the lever system is denoted by \( \lambda \), where \( \lambda = \frac{\alpha \gamma}{\beta \gamma} \) in accordance with Fig. 5, the kinematical equation of the control system for the distributor slider (disk) can be expressed in the following way:

\[
z - y = \frac{\alpha \gamma}{\beta \gamma} (x - y) = \lambda e
\]

(1)

Fig. 5. Auxiliary drawing to determine the kinematical interrelations in the actuating system for the slider of the hydraulic servomechanism
The equation of liquid flow in the hydraulic servomechanism towards the chambers of the servomechanism cylinder has the form as below:

\[
Q_A = We = \frac{W}{\lambda}(z - y) = F \frac{dy}{dt} + \frac{V_t}{B} \frac{dp_A}{dt}
\]

\[
Q_B = -We = \frac{W}{\lambda}(z - y) = -F \frac{dy}{dt} + \frac{V_t}{B} \frac{dp_B}{dt}
\]

where:

- \( Q_A \) and \( Q_B \) – flow intensity towards the A and B chambers of the cylinder in the servomechanism,
- \( W \) – amplification of the hydraulic servomechanism,
- \( p_A \) and \( p_B \) – pressure values in the A and B chambers of the servo cylinder,
- \( \lambda \) – module of volumetric elasticity of the pressure liquid,
- \( \lambda \) – position of the lever mechanism,
- \( F \) – effective area of the servo cylinder,
- \( V_t \) – volume of the servo cylinder when the piston is in its central position.

On the basis of the equations for the liquid flow (2) the following formula for the specific type of the hydraulic servomechanism (hydraulic amplifier) can be derived with use of the operational notation:

\[
\frac{We}{\lambda} = Fsy(s) + p_t \left( \frac{V_t}{2B} s + \frac{Q_o}{p_t} \right)
\]

where:

- \( Q_o \) – volumetric flow intensity through cross-section of the ports of the separator when the pressure differential is \( \Delta p = p_t/2 \),
- \( p_t \) – pressure at the input port of the servo cylinder.

For the considered steering system, the following equation for acting forces can be written:

\[
(p_A - p_B)F = C_r y(t) + C_s y(t) + \chi \frac{dy}{dt} + m \frac{d^2 y}{dt^2}
\]

where:

- \( C_r \) and \( C_s \) – factors of proportionality that bind forces and travel distances, respectively for the separator and the cylinder,
- \( m \) – load attached to the cylinder piston rod,
- \( \chi \) – viscosity friction factor that represents resistance of individual components of the system.

After Laplace’s transformation applied to the equation (4) the following form is obtained:

\[
(p_A - p_B)F = C_r y(s) + C_s y(s) + \chi s y(s) + ms^2 y(s)
\]
Substitution of (5) into (3) leads to the subsequent relationship:

$$ [x(s) - y(s)] \frac{W}{\lambda} = F s y(s) + \frac{y(s)}{F} (C_r + C_s + \chi s + ms^2) \left( \frac{V}{2B} s + \frac{Q_0}{p_t} \right) $$

Thus the adequate transformations make it possible to obtain the reverse transmittance of the system in the following form:

$$ H(s) = \frac{x(s)}{y(s)} = 1 + \frac{\lambda Q_0}{WF p_t} (C_r + C_s) + \frac{\lambda F}{W} \left[ 1 + \frac{(C_r + C_s) V}{2BF^2} + \frac{\chi Q_0}{p_t} \right] s + $$

$$ + \frac{\lambda}{WF} \left( \frac{\chi V}{2B} + \frac{m Q_0}{p_t} \right) s^2 + \frac{\lambda m V}{2B WF} s^3 $$

(6)

If pulsation \((j \omega)\) is substituted into the equation (6) instead of the Laplace’s operator, the spectral transmittance of the system can be obtained after relevant transformations. The spectral transmittance is expressed by the formula:

$$ H_1(j \omega) = \frac{x(j \omega)}{y(j \omega)} = 1 + a_1 + a_2 = \frac{\lambda}{WF} \left( \frac{\chi V}{2B} + \frac{m Q_0}{p_t} \right) \omega^2 + $$

$$ + j \left[ \frac{\lambda F}{W} \left( 1 + b_1 + b_2 + b_3 \right) \omega + \frac{\lambda m V}{2B WF} \omega^3 \right] $$

(7)

whereas:

$$ a_1 = \frac{\lambda Q_0 C_r}{WF p_t}, \quad a_2 = \frac{\lambda Q_0 C_s}{WF p_t}, $$

$$ b_1 = \frac{V C_r}{2BF^2}, \quad b_2 = \frac{V C_s}{2BF^2}, \quad b_3 = \frac{\chi Q_0}{F^2 p_t}. $$

The equation for transmittance of the system, as derived above, serves as a background for analysis of the system stability by means of enhanced linear methods, where the Nyquist criterion is the most suitable approach to evaluate stability of the system on the basis of the already known Nyquist graph (amplitude vs. phase diagram). Within the proposed method, checking of the system stability consists in drawing the hodograph \(H_1(j \omega)\) on the complex plane and examining the rotation angle of the radius-vector during its movements from the point \(\omega = 0\) to the point \(\omega = +\infty\). The system is stable when the increase of the vector argument equals zero. It means that the hodograph cannot include the point \((-1, j0)\).

If the hodograph \(H_1(j \omega)\) intersects the real axis in the point \((-1, j0)\), the system is on the edge of stability. The coefficients that occur in the equation (7) are the complementary values to the value of one, both in the real and imaginary parts.

Values of the coefficients for the equation (7) were calculated for both the longitudinal and transverse steering system of the Su-22 aircraft. On the background of design
dimensions of the hydraulic servomechanisms (hydraulic amplifiers) BU-250 and BU-220 as well as the measurement results the following values were adopted for calculations:  
- minimum supplying pressure of the hydraulic amplifier $p_{min} = 10 \text{ MPa}$,  
- amplification of the hydraulic servomechanism $W = 3.5 \text{ m}^2/\text{s}$,  
- effective area of piston faces inside the servomechanism cylinder $F = 0.0125 \text{ m}^2$,  
- factor of proportionality – ratio of the acting force and travel of the distributor disk $C_r = 1950 \text{ N/m}$,  
- factor of proportionality – ratio of the acting force and travel of the piston rod of the hydraulic cylinder $C_s = 13500 \text{ N/m}$,  
- volumetric flow intensity down the separator channel of the servomechanism $Q_0 = 6500 \text{ m}^3/\text{s}$,  
- volume of the servo cylinder when the piston is in its central position $V_t = 0.0047 \text{ m}^3$,  
- module of volumetric elasticity of the pressure liquid, specified on the background of the manual [5], but with account to volumetric efficiency of the system $B = 9.46\times10^8 \text{ N/m}^2$,  
- resistance factor due to viscosity friction in the system components $\chi = 913.3 \text{ Ns/m}$,  
- transmission ratio for the feedback module $\lambda = 2.8$.  

On the basis on the aforementioned values it was possible to calculate coefficients for the equation (7). As the results of calculations the following values were obtained:  
$a_1 = 0.0012$, $a_2 = 0.0058$, $b_1 = 0.0032$, $b_2 = 0.0019$ and $b_3 = 0.0297$. The sum of equation (7) coefficients, both their real and imaginative parts, does not exceed 3.5% of the major element, hence they can be omitted with no detriment to practical applicability of the equation (7) to analysis of the system stability. After having ignored the coefficients in the equation (7), the equation itself can be simplified and the Nyquist transmittance (amplitude vs. phase characteristic) takes the following form:

$$
H(j\omega) = \frac{x(j\omega)}{y(j\omega)} = 1 - \frac{\lambda}{WF} \left( \frac{\dot{X}_t}{2B} + \frac{mQ_t}{p_t} \right) \omega^2 + j \left[ \frac{\lambda F}{W} \omega^2 - \frac{\lambda mV_t}{2BWF} \omega^3 \right] 
$$  

where the real part $U(\omega) = 1 - \frac{\lambda}{WF} \left( \frac{\dot{X}_t}{2B} + \frac{mQ_t}{p_t} \right) \omega^2$ is the abscissa, whereas the imaginative part $jV(\omega) = j \left[ \frac{\lambda F}{W} \omega - \frac{\lambda mV_t}{2BWF} \omega^3 \right]$ is the ordinate on the complex plane $U(\omega)$, $jV(\omega)$.  

4. Nyquist hodographs on complex planes  

Using the equation (8) and the values of parameters that are listed in the preceding paragraph, namely: amplification of the hydraulic servomechanism $W$, effective area of piston faces inside the servomechanism cylinder $F$, factor of proportionality – ratio of the
acting force and travel of the distributor disk $C$, as well as ratio of the acting force and travel of the piston rod of the hydraulic cylinder $C$, volumetric flow intensity down the separator channel of the servomechanism $Q_o$, volume of the servo cylinder when the piston is in its central position $V$, module of volumetric elasticity of the pressure liquid $B$, viscosity friction factor in the system components $\chi$ as well as transmission ratio for the feedback module $\lambda$, made it possible to draw Nyquist graphs (amplitude vs. phase characteristic curves) for four values of supplying pressure at the input of separator in the hydraulic servomechanism.

The Nyquist graphs (amplitude vs. phase characteristic curves) for the longitudinal steering system of the Su-22 aircraft and for four values of supplying pressure at the input of separator in the hydraulic servomechanism are shown on Fig. 6 – 9. Fig. 6 presents the Nyquist hodograph where the supplying pressure at the input of separator in the hydraulic servomechanism was $p_t = 12.0$ MPa. The graph on Fig. 6 implies that the servo-assisting system for aircraft steering is in unstable condition as the hodograph encapsulates the point (-1, j0), i.e. increase of the radius-vector argument is negative. Fig. 7 presents the Nyquist hodograph where the supplying pressure at the input of the separator in the hydraulic servomechanism was $p_t = 15.5$ MPa. The graph on Fig. 7 implies that the servo-assisting system for aircraft steering is on the edge of stability as the hodograph line goes through the point with co-ordinates $U = -1, jV = 0$. Therefore stability of the system can be achieved for pressure values higher than $p_t = 15.5$ MPa. Fig. 8 presents the Nyquist hodograph where the supplying pressure at the input of separator in the hydraulic servomechanism was $p_t = 21.0$ MPa. The graph on Fig. 8 implies that the servo-assisting system for aircraft steering is in the stable state as the hodograph line intersects the X-axis within the interval (-1, j0) < $H_j(\omega) < (0,j0)$, therefore increment of the radius-vector argument equals to zero. Fig. 9 presents the Nyquist hodograph where the supplying pressure at the input of separator in the hydraulic servomechanism was $p_t = 30.0$ MPa. The graph on Fig. 9 implies that the servo-assisting system for aircraft steering is on the edge of stability as the hodograph line goes through the point with co-ordinates $U = -1, jV = 0$. Therefore stability of the system can be achieved for pressure values less than $p_t = 30.0$ MPa.

All the above analysis of hodograph behaviour enables to make the conclusion that the stability of the servo-assisted steering system of the Su-22 aircraft shall be preserved when the feeding pressure falls into the range from 15.5 MPa to 30.0 MPa. Pressure in the servo-assisted steering system of the Su-22 aircraft depends on aerodynamic thrust onto tail plane of the elevator. To assure correct operation of the steering system of the Su-22 aircraft the feeding pressure of the hydraulic servomechanism (hydraulic amplifier) cannot be less than 15.5 MPa. The upper limit for feeding pressure of the hydraulic servomechanism (hydraulic amplifier) is restricted by the safety valve that is set to the value of 25.5 MPa.
Fig. 6. The Nyquist graph (amplitude vs. phase characteristic curve) for the hydraulic servo-assisted steering system of the Su-22 aircraft at the pressure $p_t = 12.0\ MPa$

Fig. 7. The Nyquist graph (amplitude vs. phase characteristic curve) for the hydraulic servo-assisted steering system of the Su-22 aircraft at the pressure $p_t = 15.5\ MPa$
Fig. 8. The Nyquist graph (amplitude vs. phase characteristic curve) for the hydraulic servo-assisted steering system of the Su-22 aircraft at the pressure $p_t = 21.0$ MPa

Fig. 9. The Nyquist graph (amplitude vs. phase characteristic curve) for the hydraulic servo-assisted steering system of the Su-22 aircraft at the pressure $p_t = 30.0$ MPa
Fig. 10 and 11 present exemplary timings for pressure in the servo-assisted steering system of the Su-22 aircraft during switching the system over, the first graph refers to a new system at the beginning if its service, the second one reflects condition of the servo-assisted system after 1,500 hours of exploitation.

Fig. 10. Exemplary timings for pressure in the servo-assisted hydraulic steering system of the Su-22 aircraft during switching the system over and at the beginning if its service

Fig. 11. Exemplary timings for pressure in the servo-assisted hydraulic steering system of the Su-22 aircraft during switching the system over and after expiring 1,500 hours of its exploitation.
Timing diagrams for the servo-assisted steering system of the Su-22 aircraft show that at the beginning of the service lifetime (see Fig. 10) the pressure in the system falls into the range from 20.9 MPa to 21.3 MPa when no movements of the tail plane of the elevator are performed. Thus, the pressure is higher than the lower limit that determines stability of the system. Similarly, the timings for the servo-assisted steering system of the Su-22 aircraft at the beginning of the service lifetime but at full deflection of the tail plane of the elevator may range from 19.2 MPa to 19.8 MPa so they are again above the lower limit of stability. Not worse in case of timings for the servo-assisted steering system of the Su-22 aircraft after 1,500 hours of service (see Fig. 11). The drawing proves that in case when no movements of the tail plane of the elevator are performed the pressure in the system matches the area of stability as it varies from 19.2 MPa to 19.7 MPa. Problems may arise for servo-assisted steering systems of the Su-22 aircrafts after 1,500 hours of service at full deflection of the tail plane. System pressure drops down to the interval from 15.0 MPa to 16.2 MPa hence it is on the edge of the lower limit that determines stability of the system. It was also confirmed by experiments. When pressure in the servo-assisted steering system was decreased below 15.0 MPa vibrations of the servo-assisted system were visible.

5. Conclusions

Stability of the hydraulic system for servo-assisted steering system of the Su-22 aircraft significantly depends on technical condition of the system itself as well as the operating regime of the aircraft. The equation (8) implies that any decrease of elasticity module of the pressure liquid results in large values of the coefficient that goes with \( \omega^3 \) becomes quite large and the ordinate for the working point on the co-ordinate plane becomes negative. Hence the system becomes unstable. Reduction of the elasticity module of the pressure liquid may result from aeration of the liquid in the system (exploitation factor) or from significant decrease of volumetric efficiency of the hydraulic system (technical condition). Therefore the technical condition of the servo-assisted steering system of the Su-22 aircraft can be evaluated by examination of its stability, which assures safe on-board operation of the module. The formula for transmittance of the hydraulic system for servo-assisted steering of the aircraft that is derived in this paper serves as a background for further analysis of stability by means of enhanced linear methods. To enable the evaluation of the hydraulic system’s stability on the basis of the already-known Nyquist graph (amplitude vs. phase characteristic curve) the Nyquist criterion was used. In accordance to this criterion, checking of the system’s stability consists in drawing the hodograph \( H_1(j\omega) \) on the complex plane and examining the rotation angle of the radius-vector during is travel down the hodograph from the point \( \omega = 0 \) up to the point \( \omega = +\infty \). The system is stable if the increment of the radius-vector argument equals to zero. It means that the hodograph cannot circle around the point (-1, j0). When the hodograph \( H_1(j\omega) \) intersects the real axis in the point (-1, j0) the system is on the edge of stability and may oscillate. Analysis of the Nyquist graphs (amplitude vs. phase characteristic curves) shows that stability of the hydraulic system for servo-assisted steering of the Su-22 aircraft is preserved for the range of supplying pressure from 15.5
MPa to about 30.0 MPa. This implies that the hydraulic system for servo-assisted steering of the Su-22 aircraft cannot be operated with pressure values less than 15.5 MPa.

Analysis of the stability by means of enhanced linear methods has been used for evaluation of the technical condition of the hydraulic system for servo-assisted steering of the Su-22 aircraft during the MTBR tests. The results derived from the theoretical analysis were confirmed by the overhaul life test of the hydraulic system for servo-assisted steering of the Su-22 aircraft.

References


