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APPLICATION OF FUZZY FAULT TREE IN RISK ANALYSIS OF COLLECTIVE WATER SUPPLY SYSTEMS

ZASTOSOWANIE ROZMYTYCH DRZEW NIEZDATNOŚCI W ANALIZIE RYZYKA SYSTEMÓW ZBIOROWEGO ZAOPATRZENIA W WODĘ

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Abstract*: Application of the theory of using fuzzy fault trees is the first attempt to be used in collective water supply systems safety assessment in Poland. The paper presents a method of fault tree using fuzzy set theory. Fuzziness of basic logic gates used in the fault tree method is presented. The paper contains an application example of the method.*

Keywords: *fault tree, risk, water supply system*

Streszczenie: *Aplikacja teorii zbiorów rozmytych w metodzie drzewa niezdatność jest pierwszą próbą jej zastosowania do analizy zarządzania bezpieczeństwem systemów zbiorowego zaopatrzenia w wodę w Polsce. Artykuł przedstawia metodę rozmytego drzewa niezdatności. Zaprezentowano rozmytość podstawowych bramek logicznych z metody drzew niezdatności. Przedstawiono przykład aplikacyjny zaproponowanej metody.*

Słowa kluczowe: *drzewo niezdatności, ryzyko, system zaopatrzenia w wodę*

1. Introduction

Reliability analysis of collective water supply systems (CWSS) is currently a widely discussed topic. Despite increasingly stringent water quality standards and the obligation to monitor water quality, primary and secondary contamination of drinking water are phenomena that can not be completely eliminated. In addition, ageing of water supply networks, with inappropriate regeneration strategies, is the cause of more frequent water pipes failures [10]. The results of these events are the lack of water, limitation of water supply or poor quality water delivered to customers. In most methods to analyse and assess risk associated with the reliability of CWSS components it is required to determine the probability of the occurrence of undesirable events and their consequences [9]. If the database on the probability is inadequate in quantity (too small statistical set) or inadequate in terms of quality, the task is complicated and even impossible to solve, or accepted values are subject to a big error. This problem can be considered using the theory of fuzzy sets, in which on the basis of incomplete knowledge one can determine appropriate parameters. The probability can also be given as a fuzzy value, especially when it is estimated and not strictly defined, e.g. on the basis of data given by a number of experts [1, 2, 3, 5, 12].

The main purpose of this work is to present the application of fuzzy set theory in the CWSS reliability analysis performed by the fault tree method. In the work the fault tree method using the fuzzy set theory was shown. The process of fuzzification of the basic logic gates used in the fault tree method and the method of the determination of membership function parameters, were presented. The method to determine the validity of the elementary events of the fault tree, was shown.

2. The fault tree method

Fault tree analysis (FTA) deals with the identification of the conditions and factors that cause, may cause or contribute to the occurrence of a given top event. The fault tree is a model that describes the relationship between failures of the elementary parts of the system, operator's errors and the event associated with a particular system functional failure. The fault tree is a graphical representation of the so-called Boolean algebra [4, 6, 7].

Fault tree analysis involves the following stages:

- a description of the system and boundary conditions,
- a choice of the top event,
- constructing the tree,
- identification of the minimal cut sets for the tree,

- qualitative analysis,
- quantitative analysis.

In drawing up the tree we use the so-called functors (logic gates) specifying, among others, the logical product of events and the logical sum of events.
The probability of failure of AND gate is calculated from the formula:

$$P = \prod_{i=1}^{n} P_i \tag{1}$$

The probability of failure of OR gate is calculated from the formula:

$$P = 1 - \prod_{i=1}^{n} (1 - P_i) \tag{2}$$

Figure 1 shows the basic logic gates of FTA method.

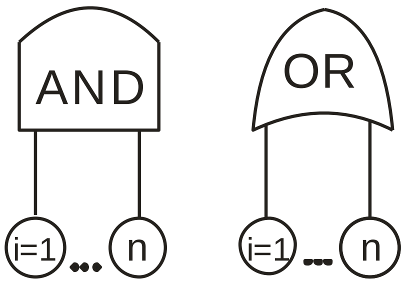

Fig. 1. Basic logic gates of FTA method

3. The concept and properties of fuzzy sets

Fuzzy sets can be used to describe different linguistic terms associated with risk analysis (small, medium, large, very large). A linguistic variable is a variable that characterizes fuzzy, imprecise terms, expressed in words, for example: about number 1, high risk, low risk value.
Before starting the process of fuzzification, the domain of discourse should be strictly defined, the so called X universe, which is an ordinary not fuzzy set. The basic properties of fuzzy sets are [8]:

- a fuzzy set is determined if all the membership functions for each element of the universe are given,
- the membership function (matching degree, compliance) assigns to each element x of the domain of discourse (space) a value within the range [0, 1], μ_A: X $\rightarrow$[0, 1], which means that every element x of the space X belongs to a fuzzy set A with certain degree of membership,

- a fuzzy set A is defined as: $A = \{\mu_A(x), x\}$,
- the values of the membership function μ_A are real numbers in the interval [0, 1],
- if $\mu_A = 0$, then variable x does not belong to the set A,
- if $\mu_A = 1$, then variable x completely belongs to the set A,
- if $0 < \mu_A < 1$, then variable x partially belongs to the set A,
- the numeric value of the membership function is called the degree of membership that can be determined by a functional dependence or in a discreet way.

The membership function can have different shapes, the most often used are Gaussian, triangular or trapezoidal functions.
In this study the triangular and trapezoidal fuzzy functions were used. The triangular fuzzy functions are described by the membership function t as:

$$\mu_A = \begin{cases} 0 & \text{dla} \quad x \le a \\ \dfrac{x-a}{b-a} & \text{dla } a < x \le b \\ \dfrac{c-x}{c-b} & \text{dla } b < x \le c \\ 0 & \text{dla} \quad x > c \end{cases} \tag{3}$$

The trapezoidal fuzzy functions are described by the membership function t as:

$$\mu_A = \begin{cases} 0 & \text{dla} \quad x \le a \\ \dfrac{x-a}{b-a} & \text{dla } a < x \le b \\ 1 & \text{dla } b < x \le c \\ \dfrac{d-x}{d-c} & \text{dla } c < x \le d \\ 0 & \text{dla} \quad x > d \end{cases} \tag{4}$$

where:
x – variable,
μ_A – the membership function of variable x in the fuzzy set A,
a, b, c, d – the membership function parameters.

4. The process of the logic gates fuzzification

Fuzzy AND gate

For the fuzzy gate FAND the following relation is met:

$$FAND(p_1, p_2, ..., p_n) = \prod_{i=1}^{n} p_i \tag{5}$$

– For the triangular membership function, characteristics of a set is described by (3), using the membership function (a, b, c). For this function:

$$FAND(p_1, p_2, ..., p_n) = \prod_{i=1}^{n} (a, b, c) = (\prod_{i=1}^{n} a_i, \prod_{i=1}^{n} b_i, \prod_{i=1}^{n} c_i) \tag{6}$$

– For the trapezoidal fuzzy sets, characteristics of a set is described by (4), using the membership function (a, b, c, d). Consequently, there was obtained:

$$FAND(p_1, p_2, ..., p_n) = \prod_{i=1}^{n} (a, b, c, d) = (\prod_{i=1}^{n} a_i, \prod_{i=1}^{n} b_i, \prod_{i=1}^{n} c_i, \prod_{i=1}^{n} d_i) \tag{7}$$

Fuzzy OR gate

For the fuzzy gate FOR the following relation is met:

$$FOR(p_1, p_2, ..., p_n) = 1 - \prod_{i=1}^{n} (1 - p_i) \tag{8}$$

– For the triangular membership function, characteristics of a set is described by (3), using the membership function (a, b, c). For this function:

$$\begin{aligned} FOR(p_1, p_2, ..., p_n) &= 1 - \prod_{i=1}^{n} [1 - (a, b, c)] = 1 - \prod_{i=1}^{n} (1 - a, 1 - b, 1 - c)) = \\ &= 1 - (\prod_{i=1}^{n} (1 - a_i), \prod_{i=1}^{n} (1 - b_i), \prod_{i=1}^{n} (1 - c_i)) = (1 - \prod_{i=1}^{n} (1 - a_i), 1 - \prod_{i=1}^{n} (1 - b_i), 1 - \prod_{i=1}^{n} (1 - c_i)) \end{aligned} \tag{9}$$

– For the trapezoidal fuzzy sets, characteristics of a set is described by (4), using the membership function (a, b, c, d). Consequently, there was obtained:

$$\begin{aligned} FOR(p_1, p_2, ..., p_n) &= 1 - \prod_{i=1}^{n} [1 - (a, b, c, d)] = \\ &= (1 - \prod_{i=1}^{n} (1 - a_i), 1 - \prod_{i=1}^{n} (1 - b_i), 1 - \prod_{i=1}^{n} (1 - c_i), 1 - \prod_{i=1}^{n} (1 - d_i)) \end{aligned} \tag{10}$$

5. Determination of membership function parameters

Assumption

Prior to the analysis, the values of membership function parameters must be determined, that is, for the triangular function (a, b, c), for the trapezoidal function (a, b, c, d). If the parameter values are determined by n experts, one final value should be selected for calculations.

Determination of membership function parameters for the triangular membership function

The elementary event is described by the triangular functions $X_i=(b_i-\alpha_i, b_i, b_i+\alpha_i)$, where:

- n - a number of experts,
- i - one particular expert out of n experts,
- b_i - a value of the b parameter of the triangular membership function, according to the i-th expert,
- α_i - a value of deviation of the b parameter from the membership function limit values, according to the i-th expert.

The function which describes the decisions of experts in the best way is:

$$E = (e - \varepsilon,\ e,\ e + \varepsilon) \tag{11}$$

where:

- e - a value of the parameter b of the triangular membership function that best describes the choice of experts
- ε - e parameter deviation from the membership function limit values that best describes the choice of experts

The values e and ε will be obtained if the fuzzy set E has the smallest deviation from any X_i. The resulting set will be still the triangular fuzzy set, but with a smaller span. According to [11] the sum of squared deviations from the mean was proposed:

$$S = \sum_{i=1}^{n} (2(\varepsilon - \alpha_i))^2 \tag{12}$$

which achieves the minimum value for:

$$\varepsilon = \frac{1}{n} \sum_{i=1}^{n} \alpha_i \tag{13}$$

If Y denotes the absolute deviation:

$$Y = \max\left|e - b_i\right| \tag{14}$$

then Y achieves the minimum for:

$$e = \frac{\min b_i + \max b_i}{2} \tag{15}$$

Determination of membership function parameters for the trapezoidal membership function

For the trapezoidal fuzzy sets $X_i=(b_i-\alpha_i, b_i, c_i, c_i+\alpha_i)$ the function that best describes the decisions of experts is $E=(e-\varepsilon, e, f, e+\varepsilon)$. Just as for the triangular functions, for the trapezoidal fuzzy sets the following relations are met:

$$\varepsilon = \frac{1}{n}\sum_{i=1}^{n}\alpha_i \tag{16}$$

$$e = \frac{\min b_i + \max b_i}{2} \tag{17}$$

$$f = \frac{\min c_i' + \max c_i'}{2} \tag{18}$$

where:

b_i - a value of the parameter b of the trapezoidal membership function, according to the i-th expert

c_i - a value of the parameter c of the trapezoidal membership function, according to the i-th expert

α_i - a value of deviation of parameters b and c from the membership function limits, according to the i-th expert

e - a value of the parameter b of the trapezoidal membership function that best describes the choice of experts

f - a value of the parameter c of the trapezoidal membership function that best describes the choice of experts.

ε - e and f parameters deviation from the membership function limits that best describes the choice of experts

6. Application example

Figure 2. presents the analysed fault tree which shows the cause-effect relationship for undesirable event in CWSS.

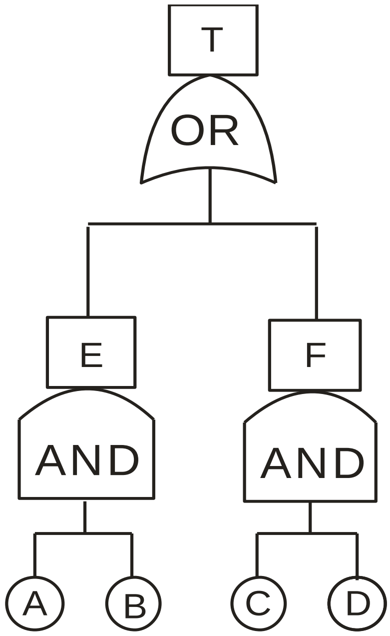

Fig. 2. Basic logic gates of FTA method

where:

A - an elementary event - contaminated water in source
B - an elementary event - inadequate water treatment
C - an elementary event - pipe break
D - an elementary event - failure do not repaired immediately
E - an intermediate event - contamination in water distribution subsystem
F - an intermediate event – water do not delivered
T - a top event involving threat to health of water consumers

The whole FTA tree is described using Boolean algebra. Event T can be establish according to equations:

$$T=E\cup F,\ E=A\cap B,\ F= C\cap D$$

Prior to the analysis four experts were asked to determine the membership function parameters for the elementary events - for the elementary events A, B, C, the triangular membership function distribution was assumed, for the elementary event D, the trapezoidal membership function distribution was assumed. The survey results are shown in tab. 1.

Table 1. Survey results determining the membership function parameters for the elementary events

	Event A			Event B			Event C			Event D			
expert	a	b	c	a	b	c	a	b	c	a	b	c	d
1	0,05	0,07	0,09	0,01	0,03	0,05	0,01	0,03	0,05	0,01	0,03	0,04	0,06
2	0,04	0,07	0,10	0,02	0,04	0,06	0,01	0,02	0,03	0,02	0,03	0,04	0,05
3	0,05	0,06	0,07	0,02	0,04	0,06	0,02	0,04	0,06	0,04	0,06	0,08	0,10
4	0,03	0,06	0,09	0,03	0,07	0,11	0,03	0,04	0,05	0,07	0,09	0,010	0,12

Using the method presented above for the determination of membership function parameters we obtained 4 fuzzy sets which best describe the probability that the elementary events will occur.

A = (0,0425, 0,0650, 0,0875)
B = (0,0250, 0,0500, 0,0750)
C = (0,0150, 0,0300, 0,0450)
D = (0,0425, 0,0600, 0,0700, 0,0875)

Determined above the membership function parameters of four fuzzy sets were used to determine a fuzzy set of the probability of top event. The formulas (7) and (10) were used for the calculation and as a result we obtained:

T = (0,0017, 0,0050, 0,00534, 0,0105)

Next, the fuzzy set parameters of the probability of top event while the elementary events A, B, C or D do not occur, were determined (tab. 2):

Table 2. The fuzzy set parameters of the probability of top event

	T_i (a_i, b_i, c_i, d_i)
without A	(0,0256, 0,0517, 0,0520, 0,0786)
without B	(0,0431, 0,0667, 0,0670, 0,0911)
without C	(0,0435, 0,0631, 0,0730, 0,0935)
without D	(0,0160, 0,0332, 0,0332, 0,0513)

where:
T_i - the possibility of top event in absence of event i

Fuzzy importance index (FII) as the Euclidean distance between the membership function parameters T and Ti was determined:

$$FII(i) = \sqrt{(a - a_i)^2 + (b - b_i)^2 + (c - c_i)^2 + (d - d_i)^2} \qquad (19)$$

The figure 3 show the values of FII for all the elementary events:

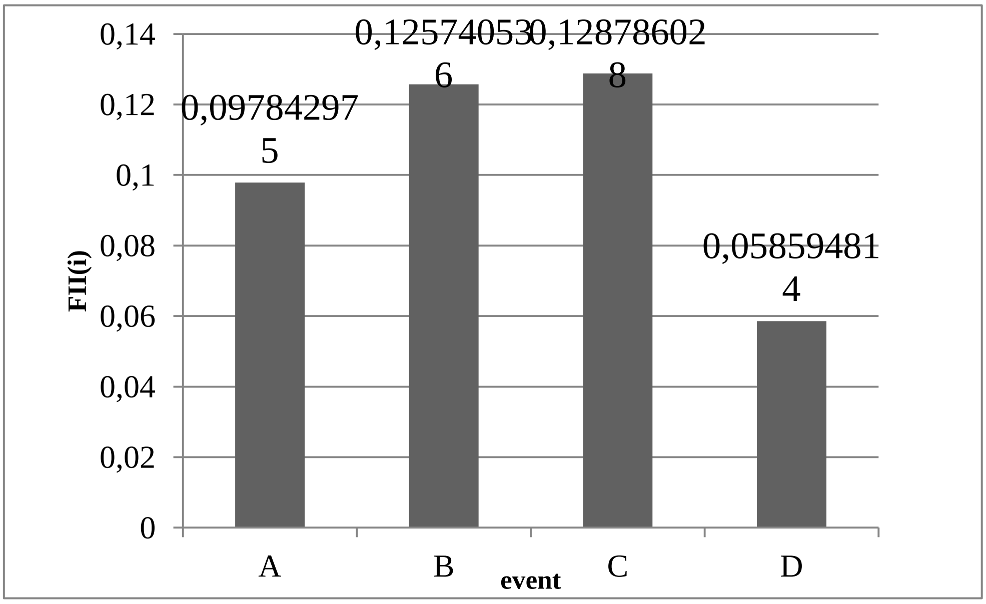

Fig. 3. The values of FII for all the elementary events

From the obtained results it can be concluded that the elementary event C has the highest value of FII, which means that the occurrence of this event has the greatest impact on the probability of the top event occurrence.

7. Conclusions

- The theory of fuzzy sets for establishing the degree of membership of a given variable in the fuzzy set significantly expands the possibilities of CWSS reliability analysis.
- The proposed method can be used if you have incomplete data on the elementary events and when it is not possible to determine the probability of their occurrence.
- The presented way of logic gates fuzzication from FTA method allows to determine the fuzzy probability of top event occurrence.

– The proposed method of determining the values of the membership function parameters based on the survey results, supplemented by the experts, provides an accurate way to determine the specific parameters of the membership function for each elementary event.

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