POSSIBILITY OF APPLICATION OF THE THEORY OF SEMI-MARKOV PROCESSES TO DETERMINE RELIABILITY OF DIAGNOSING SYSTEMS

MOŻLIWOŚĆ ZASTOSOWANIA TEORII PROCESÓW SEMI-MARKOWA DO OKREŚLENIA NIEZAWODNOŚCI SYSTEMÓW DIAGNOZUJĄCYCH

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Abstract: *The paper provides justification for the necessity to define reliability of diagnosing systems (SDG) in order to develop a diagnosis on state of any technical mechanism being a diagnosed system (SDN). It has been shown that the knowledge of SDG reliability enables defining diagnosis reliability. It has been assumed that the diagnosis reliability can be defined as a diagnosis property which specifies the degree of recognizing by a diagnosing system (SDG) the actual state of the diagnosed system (SDN) which may be any mechanism, and the conditional probability $p(S^*/\boldsymbol{K}^*)$ of occurrence (existence) of state S^* of the mechanism (SDN) as a diagnosis measure provided that at a specified reliability of SDG, the vector $\boldsymbol{K}^*$ of values of diagnostic parameters implied by the state, is observed. The probability that SDG is in the state of ability during diagnostic tests and the following diagnostic inferences leading to development of a diagnosis about the SDN state, has been accepted as a measure of SDG reliability. The theory of semi-Markov processes has been used for defining the SDG reliability, that enabled to develop a SDG reliability model in the form of a seven-state (continuous-time discrete-state) semi-Markov process of changes of SDG states.*

Keywords: *decision, diagnostics, probability, reliability, diagnosing system*

Streszczenie: *W opracowaniu uzasadniono konieczność określenia niezawodności systemów diagnozujących (SDG) do sformułowania diagnozy o stanie dowolnego urządzenia technicznego jako systemu diagnozowanego (SDN). Wykazano, że znajomość niezawodności SDG umożliwia określenie wiarygodności diagnozy. Przyjęto, że wiarygodność diagnozy może być określona jako właściwość diagnozy określająca stopień rozpoznania przez system diagnozujący (SDG) rzeczywistego stanu systemu diagnozowanego (SDN), którym może być dowolne urządzenie, zaś za miarę diagnozy przyjęto prawdopodobieństwo warunkowe $p(S^*/\boldsymbol{K}^*)$ pojawienia się (istnienia) stanu S^* urządzenia (SDN) pod warunkiem, że obserwowany jest wektor $\boldsymbol{K}^*$ wartości parametrów diagnostycznych implikowanych przez ten stan, przy określonej niezawodności SDG. Zaś za miarę niezawodności SDG przyjęto prawdopodobieństwo tego, że w czasie badań diagnostycznych i kolejnych wnioskowań diagnostycznych prowadzących do opracowania diagnozy o stanie SDN, system SDG będzie w stanie zdatności. Do określenia niezawodności SDG zastosowano teorię procesów semi-Markowa, która umożliwiła opracowanie modelu niezawodnościowego SDG w formie siedmiostanowego procesu semimarkowskiego (dyskretnego w stanach i ciągłego w czasie) zmian stanów SDG.*

Słowa kluczowe: *diagnostyka, prawdopodobieństwo, niezawodność, system diagnozujący*

1. Introduction

A reasonable operation of mechanisms requires making the right decisions that are appropriate for the current operating situation. An opportunity to make right decisions exists when the reliability of the diagnosis on the technical state of the operated mechanisms is known and it is possible to identify the consequences (c) of making the given decision [1, 2, 3, 4]. An assessment of the reliability of the diagnosis on the technical condition of each mechanism (as *SDN* - diagnosed system) is possible only if the reliability of the appropriate diagnosing system (*SDG*) is known, which is necessary to develop a diagnosis on the technical state of *SDN*. The probability of the system's correct work during tests and diagnostic inference that results in a diagnosis, can be taken as a measure of *SDG* reliability. This probability can be determined by applying the theory of semi-Markov processes. The requirement to determine the mentioned probability can be justified wider by describing the importance of the *SDG* reliability for making operating decisions.

2. Interpretation for diagnosis reliability

For making operating decisions we need among others the knowledge of diagnosis reliability $p(S^*/\mathbf{K}^*)$ about the technical state of the mechanism, whose the measure can be the probability of occurrence of the state s_1^* or s_2^*.
In extreme cases:
- reliable (fully reliable) diagnosis can be assigned with a digit 1, which should be considered that the diagnosis is entirely reliable, i.e. right;
- unreliable diagnosis can be assigned with a digit 0, which should be considered that the diagnosis is entirely unreliable, i.e. wrong.

In the operating practice, such an alternative assessment of diagnosis reliability, consisting in assigning it with a digit 1 or 0, is insufficient. Therefore, the papers [4, 5] present a proposal to determine diagnosis reliability in the form of the probability $P(S^*/\mathbf{K}^*)$, where S^* – state of mechanism, $\mathbf{K}^*$ – vector of values of diagnostic parameters characteristic for state S^*. The papers provide an assumption that in the evaluative sense the diagnosis reliability as a diagnosis property, can be determined by the values of important in certain cases indexes characterizing the degree of recognizing by SDG the state of SDN (mechanism), so this may be understood as the conditional probability $P(S^*/\mathbf{K}^*)$, logical probability $P_L(S^*)$ or statistical probability $P_S(S^*)$ [9].

The paper [4, 5] submits a proposal of formulas as the measures of the diagnosis reliability and accuracy. For deriving the formulas there were applied: a conditional probability formula for the events A_1, S^*, $\mathbf{K}^*$ and a continuous-time discrete-state semi-Markov model of the process of using *SDG* $\{W(t): t \geq 0\}$ [4, 5], where:

- A_1 - event representing a proper operation of *SDG* during development of the diagnosis,

- S^* - event representing an occurrence of state S^* of the mechanism (*SDN* – diagnosed system),
- $\mathbf{K}^*$ - event indicating an occurrence of a particular vector $\mathbf{K}^*$ of values of diagnostic parameters as a result of occurring the state S^* of *SDN*.

In consequence, the formula defining probability $P(S^*/\mathbf{K}^*)$ as a measure of diagnosis reliability, is obtained in the form [4, 5] as follows:

$$P(S^* / \mathbf{K}^*) = \frac{P(A_1)P(S^*)P(\mathbf{K}^* / S^*)}{P(\mathbf{K}^*)P(A_1 / \mathbf{K}^* \cap S^*)} \tag{1}$$

For a reliable *SDG* (so such *SDG* for which $P(A_1) = 1$ exists) the formula (1) takes the form as follows:

$$P(S^* / \mathbf{K}^*) = \frac{P(S^*)P(\mathbf{K}^* / S^*)}{P(\mathbf{K}^*)} \tag{2}$$

which is the measure of the diagnosis accuracy [4, 5].

From the formula (2) it follows that even when *SDG* works reliably during diagnostic tests and then during diagnostic inference until the diagnosis on *SDN* technical state is developed, the diagnosis cannot be certain. This results from the fact that while developing a diagnosis (inference) about the state S^* of *SDN*, the statement $\mathbf{K}^*$, so the statement that just this and not any other vector ($\mathbf{K}^*$, in this case) of values of diagnostic parameters was recorded by *SDG*, is regarded to be completely certain premise. While the sentence S^*, saying that just this and not any other state (S^*, in this case) of *SDN*, is an inference developed on the basis of the statement $\mathbf{K}^*$, which is the result of completed non-deductive inference. Therefore, identification of the technical state of *SDN* consists in developing the hypothesis: *SDN is in state S^* because vector $\mathbf{K}^*$ of the values of diagnostic parameters is observed.* In this case this inference is a reductive inference, so proceeds according to the following scheme [9]:

$$\mathbf{K}^*, S^* \Rightarrow \mathbf{K}^* \vdash S^* \tag{3}$$

where:

$\mathbf{K}^*$ – completely certain premise,
S^* – inference developed on basis of the sentence $\mathbf{K}^*$.

In the case, when the sentence S^* is an inference being developed on the basis of the sentence $\mathbf{K}^*$ (regarded as a completely certain premise) in the process of inferring, it can be assumed that the sentence S^* is made probable by the sentence $\mathbf{K}^*$. The measure for this can be a conditional probability defined by the formula (1) if

there is no certainty that *SDG* works reliably or by the formula (2) if *SDG* works reliably during diagnostic tests and inference.

In general, there is no certainty that SDG works reliably during tests and diagnostic inference [8, 10] and therefore, it is important to determine the probability of its correct operation. For this reason, it is necessary to identify the reliability of *SDG*. This requires consideration of at least a two-state reliability model for *SDG*, thus an assumption that this can find in only two mutually exclusive states, i.e. state of ability (s_0) and state of disability ($\bar{s}_0$). However, due to the fact that in diagnostics of mechanisms (*SDN*) there are applied different methods of testing their states (and hence different diagnosing devices), it becomes necessary to distinguish instead of one state $\bar{s}_0$, a number of states of disability s_j ($j = 1, 2, ..., n$) of *SDG*, which occur in consequence of failures in diagnosing devices that belong to *SDG* used along with the taken methods for testing the *SDN* state. The further considerations on the reliability of *SDG* include the states of disability s_j, $j = 1, 2, ..., 6$ ($s_j \in \bar{s}_0$), where j – type of *SDG* disability resulting from a failure of its subsystem *SDG_j*, where the indexes $j = \overline{1,6}$ denote e.g.: 1 – subsystem for testing the acoustic emission, 2 – subsystem for visual (endoscopic) testing, 3 – subsystem for thermal (temperature, pressure) testing, 4 – subsystem for vibration testing (for NVH tests), 5 – subsystem for wear testing by employing the method of radionuclide X-ray fluorescence (*XRF*), 6 – subsystem for thermographic testing (for infrared thermal image analysis).

To determine the *SDG* reliability, at such approach to testing the reliability of *SDG*, we can apply the theory of semi-Markov processes (of continuous-time discrete-state type) and develop a seven-state semi-Markov model of the process of changes of *SDG* states (state of ability s_0 and states of disability s_j, $j = \overline{1,6}$), which is necessary to derive the formula for the probability of staying the *SDG* in state of ability (s_0), so in a state which is indispensable to develop a reliable diagnosis.

3. A semi-Markov model of the process of changes of states for diagnosing systems

Application of the semi-Markov model of the process of changes of states for a diagnosing system (*SDG*) enables consideration of its preventive maintenance service [2, 8, 10] and, therefore, consideration of *SDG* reliability state s_0, i.e. state of ability and states of disability s_j ($j = \overline{1,6}$) of its particular subsystems *SDG_j*, $j = 1, 2, ..., 6$.

The semi-Markov model of the process of changes of reliability states for a the diagnosing system (*SDG*) can therefore, be considered as a semi-Markov process $\{W(t): t \geq 0\}$ with the set of states $S = s_i$; $i = 0, 1, ... , 6$. The interpretation of the states $s_i \in S(i = 0, 1, ... , 6)$ is as follows: s_0 – state of ability of *SDG* and

simultaneously of all its subsystems SDG_j, ($j = \overline{1,6}$), s_1 – state of disability of the subsystem SDG_1 for acoustic emission testing, s_2 – state of disability of the subsystem SDG_2 for visual (endoscopic) testing, s_3 – state of disability of the subsystem SDG_3 for thermal (temperature, pressure) testing, s_4 – state of disability of the subsystem SDG_4 for vibration (NVH) testing, s_5 – state of disability of the subsystem SDG_5 for wear testing by using X-ray radionuclide fluorescence method (*XRF*), s_6 – state of disability of the subsystem SDG_6 for thermographic testing (infrared thermal image analysis). Changes of the listed states s_i ($i = 0, 1, ..., 6$) proceed at subsequent times t_n ($n \in N$), where at time $t_0 = 0$ a diagnosing system (*SDG*) is in state s_0. The state s_0 lasts until any of the distinguished subsystems SDG_j ($j = \overline{1,6}$) fails. The states $s_j (i = 1, ..., 6)$ last as long as the failed subsystem SDG_j is renovated or replaced by another one in case the renovation is found unprofitable. It can be assumed that the state of *SDG* at time t_{n+1} and the time interval of duration of the state achieved at time t_n do not depend on the states occurred at times $t_0, t_1, ..., t_{n-1}$ or the time intervals of their duration. Thus, the process $\{W(t): t \geq 0\}$ of changes of states $s_i; i = 0, 1, ... ,6$ is a semi-Markov process [4, 6]. The graph of changes of the reliability states s_j of *SDG* ($i = \overline{0,6}$) is shown in Fig. 1. The initial distribution of the process $\{W(t): t \geq 0\}$ is as follows:

$$P\{W(0) = s_i\} = \begin{cases} 1 & dla \quad i = 0 \\ 0 & dla \quad i = 1, \ 2, \ ..., \ 6 \end{cases} \tag{4}$$

whereas its matrix function (in accordance with the graph shown in Fig. 1) is of the following form:

$$\mathbf{Q(t)} = \begin{bmatrix} 0 & Q_{01}(t) & Q_{02}(t) & Q_{03}(t) & Q_{04}(t) & Q_{05}(t) & Q_{06}(t) \\ Q_{10}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{20}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{30}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{40}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{50}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{60}(t) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

The matrix function $\mathbf{Q(t)}$ is a model of changes of the reliability states of *SDG*. Non-zero elements $Q_{ij}(t)$ of the matrix $\mathbf{Q(t)}$ depend on the distributions of random variables which are the time intervals of staying the process $\{W(t): t \geq 0\}$ in states $s_i \in S(i = 0, 1,..., 6)$.

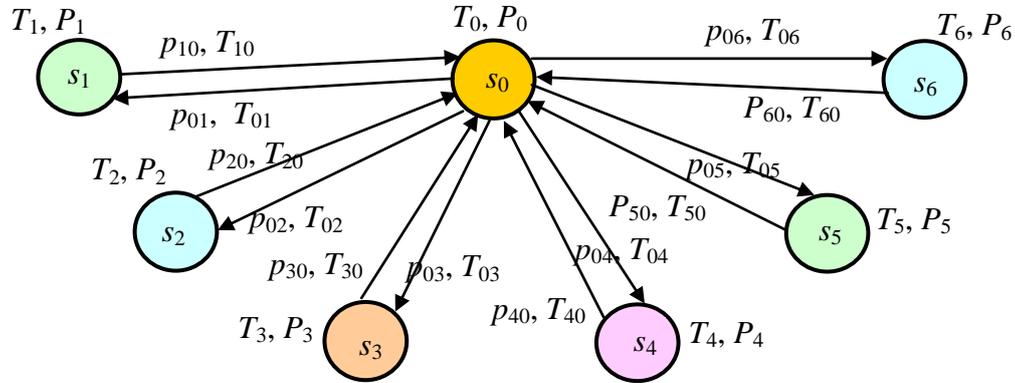*Fig. 1. Graph of changes of reliability states s_i; $i = 0, 1, ... ,6$ for a diagnosing system SDG for a crank-piston mechanism of crosshead main engine:*
s_0 – *state of full ability of SDG*, s_1 – *state of disability of the subsystem SDG for acoustic emission testing*, s_2 – *state of disability of the subsystem SDG for visual (endoscopic) testing*, s_3 – *state of disability of the subsystem SDG for thermal (temperature, pressure) testing*, s_4 – *state of disability of the subsystem SDG for vibration (NVH) testing*; s_5 – *state of disability of the subsystem SDG for wear testing by using X-ray radionuclide fluorescence method (XRF)*; s_6 – *state of disability of the subsystem SDG for thermographic testing (infrared thermal image analysis)*; T_0 – *time of duration of the state of ability s_0*; T_1 – *time of duration of the state of disability s_1*, T_2 – *time of duration of the state of disability s_2*; T_3 – *time of duration of the state of disability s_3*; T_4 – *time of duration of the state of disability s_4*; T_5 – *time of duration of the state of disability s_5*; T_6 – *time of duration of the state of disability s_6*; P_0 – *probability of staying SDG in state s_0*, P_1 – *probability of staying SDG in state s_1*, P_2 – *probability of staying SDG in state s_2*, P_3 – *probability of staying SDG in state s_3*, P_4 – *probability of staying SDG in state s_4*, P_5 – *probability of staying SDG in state s_5*, P_6 – *probability of staying SDG in state s_6*, p_{ij} – *probability of transition from state s_i to state s_j*; T_{ij} – *time of duration of state s_i providing that the subsequent state is s_j; $i,j = 0,1,2,..., 6$; $i \neq j$.*

The elements are the probabilities of transition of the mentioned process from state s_i to state s_j $(s_i, s_j \in S)$ at time not longer than t, defined as follows:

$$Q_{ij}(t) = P\{W(\tau_{n+1}) = s_j, \ \tau_{n+1} - \tau_n < t | W(\tau_n) = s_i\} = p_{ij}F_{ij}(t) \qquad (6)$$

where:

p_{ij} - probability of transition at one step in homogeneous Markov chain;

p_{ij} $= P\{Y(\tau_{n+1}) = s_j | Y(\tau_n) = s_i = \lim\limits_{t \to \infty} Q_{ij}(t)$;

$F_{ij}(t)$ - distribution function for the random variable T_{ij}, denoting the time of duration of state s_i of the process $\{W(t): t \geq 0\}$, providing that the subsequent state of the process is s_j.

Due to the matrix (5) of the process $\{W(t): t \geq 0\}$ is a stochastic matrix, the matrix of the probability of transition of the Markov chain embedded in this process is as follows [6]:

$$
\mathbf{P} = \begin{bmatrix}
0 & p_{01} & p_{02} & p_{03} & p_{04} & p_{05} & p_{06} \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\tag{7}
$$

The process $\{W(t): t \geq 0\}$ is irreducible [6, 7] and random variables T_{ij} have finite positive expected values. Therefore, its limiting distribution

$$
P_j = \lim_{t \to \infty} P_{ij}(t) = \lim_{t \to \infty} P\{W(t) = s_j\}, \, s_j \in S(j = 0, 1, ..., 6)
\tag{8}
$$

is of the following form [6]:

$$
P_j = \frac{\pi_j E(T_j)}{\sum_{k=0}^{6} \pi_k E(T_k)}
\tag{9}
$$

Probabilities $\pi_j (j = 0, 1,..., 6)$ in the formula (9) are the limiting probabilities of the embedded Markov chain.

Determination of the limiting distribution (9) requires solution of the following system of equations:

$$
\left.
\begin{aligned}
&[\pi_0, \pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6] = [\pi_0, \pi_1, \pi_2, \pi_3 \, \pi_4, \pi_5, \pi_6] \cdot \mathbf{P} \\
&\sum_{k=0}^{6} \pi_k = 1
\end{aligned}
\right\}
\tag{10}
$$

As a result of solving the system of equations (10) by using the formula (9) the following relationships can be obtained:

$$
P_0 = \frac{E(T_0)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)}, \quad P_1 = \frac{p_{01} E(T_1)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)}, \quad P_2 = \frac{p_{02} E(T_2)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)},
$$

$$
P_3 = \frac{p_{03} E(T_3)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)}, \quad P_4 = \frac{p_{04} E(T_4)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)}, \quad P_5 = \frac{p_{05} E(T_5)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)}, \tag{11}
$$

$$
P_6 = \frac{p_{06} E(T_6)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)}.
$$

The probability P_0 is a limiting probability that in longer period of operation (in theory at $t \rightarrow \infty$) *SDG* is in state s_0 ($P_0 = P(A_1)$). This probability is therefore a coefficient of the system's technical readiness for diagnosing. However, the probabilities $P_j(j = 1, 2,..., 6)$ are limiting probabilities of existing states $s_j \in$ S of the system at $t \rightarrow \infty$, i.e. the probabilities of being its subsystems SDG_j ($j = 1,2,..,6$) in states of disability, so also the whole *SDG*, due to its serial reliability structure.

An exemplary realization of the process $\{W(t): t \geq 0\}$, showing an occurrence of the reliability states of *SDG* during operation, is presented in Fig. 2.

For the operating practice of *SDG* adopted to identify the states of the diagnosed systems (*SDN*), important is also one-dimensional distribution of the process $\{W(t): t \geq 0\}$, whose elements are the functions $P_k(t)$ denoting the probability that at (any) time t the process is in state $s_k \in S(k = 0, 1, ..., 6)$. This momentary distribution can be calculated by using the initial distribution (4) of the process $\{W(t): t \geq 0\}$ and the functions $P_{ij}(t)$ being the probabilities of transition of the process from state s_i to state s_j ($s_i \in$ S, $s_j \in$ S, $i \neq j$; $i, j = 0, 1, ..., 6$). Calculation of the transition probabilities requires the knowledge of the functions $F_{ij}(t)$, i.e. distribution functions of random variables $T_{ij}(i = j; i, j = 0, 1, ..., 6)$, which are also needed to determine the functions $Q_{ij}(t)$ (with interpretation (6)), which are the elements of the matrix **Q(t)** defined by the relationship (5). Therefore, there are needed the proper reliability tests of *SDG*.

The presented reliability description of diagnosing systems (*SDG*) can, of course, be developed by specifying as many reliability states as they are essential for the operating practice of the systems, i.e. needed by a user of a given type of systems to ensure their rational operation.
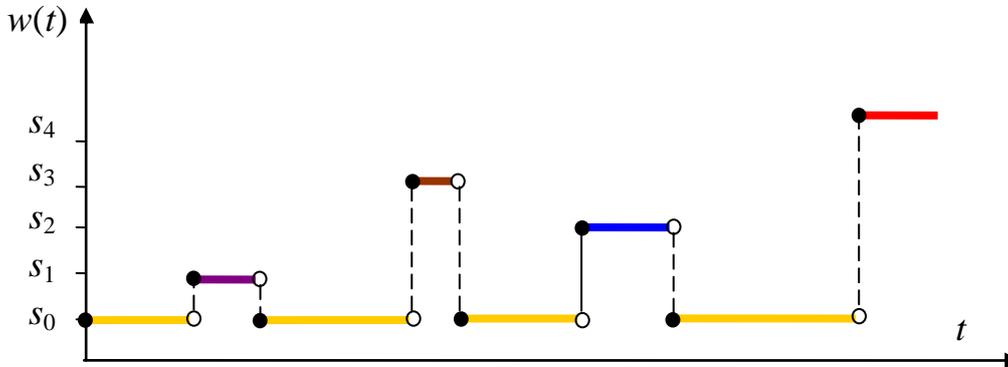
Fig. 2. Exemplary realization of the process {W(t): t ≥ 0} for a diagnosing system (SDG):

s_0 – state of full ability of SDG, s_1 – state of disability of the subsystem SDG for acoustic emission testing, s_2 – state of disability of the subsystem SDG for visual (endoscopic) testing, s_3 – state of disability of the subsystem SDG for thermal (temperature, pressure) testing, s_4 – state of disability of the subsystem SDG for vibration testing (NVH testing)

4. Remarks and conclusions

Application of the theory of semi-Markov processes for testing the reliability of diagnosing systems (*SDG*) enables to define not only the probability of staying the systems of this type in state of ability (s_0) and in particular states of disability s_j($j = \overline{1,6}$), but also the reliability of the diagnosis on the technical state of the diagnosed systems (*SDN*), which can be any mechanisms.

Semi-Markov processes are more and more often used for solving various problems in the field of reliability, mass service and diagnostics of mechanisms.

Application of the processes in the practice requires to satisfy the two conditions:
- collection of the relevant mathematical statistics;
- development of a semi-Markov model of changes of reliability states of a system with a small number of states and a simple (in the mathematical sense) matrix function **Q(t)**.

The second condition is particularly important for calculation of the momentary distribution $P_{ij}(t)$, ($i \neq j$; $i, j = 0, 1, ..., 6$) for the process of changes of reliability states {$W(t)$: $t \geq 0$} for a diagnosing system (*SDG*). As known, this distribution can be calculated when we know the initial distribution of the process {$W(t)$: $t \geq 0$} and the functions $Q_{ij}(t)$ of the matrix **Q(t)**, which are the conditional probabilities of transition of the process from one reliability state to another. Calculation of the transition probabilities $P_{ij}(t)$ consists in solving a system of Volterra integral equations of the second kind (system of equations of convolution type) [6], in which the known quantities are the elements $Q_{ij}(t)$ of the matrix function **Q(t)** for the studied process {$W(t)$: $t \geq 0$}. When the number of states of the process is small and/or its matrix function is simple, the system can be solved by applying a Laplace–Stieltjes transform. However, if the number of states of this process is

high or when at small number of states its matrix function is very complex, it is possible to obtain only an approximate solution for the system of equations. The solution (numeric) does not provide possibility to determine the probabilities of occurring the particular states of the process, when time of its duration has a large value (in theory, when t → ∞). From the theory of semi-Markov processes it results that in case of ergodic semi-Markov processes, the probabilities tend over time to strictly defined (constant) numbers. The numbers are called limiting probabilities of the states, and the sequence of the numbers makes a limiting distribution (11).

The limiting distribution enables to define a coefficient of a diagnosing system readiness for proper operation and developing a reliable diagnosis at any time [2, 6].

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