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SHIP'S HORIZONTAL PLANE DETERMINATION BY MEANS OF THREE LASER RANGE SENSORS IN PILOT NAVIGATION AND DOCKING SYSTEM

WYZNACZENIE POŁOŻENIA UMOWNEJ WODNICY STATKU ZA POMOCĄ TRZECH DALMIERZY LASEROWYCH W PILOTOWYM SYSTEMIE NAWIGACYJNO-DOKUJĄCYM

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***Abstract**: Modern ship's navigation support systems relay mainly on GNSS technology, as prime source of position. The advantages of such solution are obvious and undeniable. On the other hand in critical situations the dependency of the one system presents a great deal of risk. Taking above into consideration the idea of building GNSS independent Pilot Navigation and Docking System (PNDS) was put into practice. PNDS utilizes the range measurements between laser head and ship's side to determine the ship's horizontal plane presented on the screen. The study comprises the case in which three networked rangefinders were deployed on the berth side in a line. Such location of sensors causes uncertainty in determining of ship's outline contour and speed in relation to assigned coordinate reference system. The final algorithm presented in the article, taking into account all underdetermined conditions, has been tested in PNDS system constructed in Maritime University in Szczecin.*

***Keywords:** spline approximation, curve fitting, state vector estimation, range measurement, laser sensor, docking system, pilot system*

***Streszczenie:** Współczesne systemy wspomagania nawigacji opierają się głównie na technologii GNSS, jako podstawowym i często jedynym źródle informacji pozycyjnej. Zalety tego rozwiązania są ewidentne, jednakże w sytuacjach krytycznych pod względem bezpieczeństwa zależność od jednego systemu powoduje znaczny wzrost ryzyka. Nawigacyjny System Pilotowo-Dokujący (PNDS) został zaprojektowany z wykorzystaniem alternatywnej metody wyznaczenia położenia kadłuba podchodzącej do nabrzeża jednostki. Zastosowano w nim pomiary odległości pomiędzy głowicami laserowymi a burtą statku w celu wyliczenia położenia umownej obwiedni (konturu) kadłuba. Badania objęły przypadek trzech dalmierzy laserowych ustawionych w linii prostej na nabrzeżu. Takie usytuowanie czujników powoduje niepewność wyznaczenia położenia kadłuba i prędkości w przyjętym układzie współrzędnych. Zaprezentowany w artykule algorytm, biorąc pod uwagę przypadki niedookreślone, został przetestowany w systemie PNDS zbudowanym w Akademii Morskiej w Szczecinie.*

***Słowa kluczowe:** aproksymacja krzywymi sklejanymi, dopasowanie krzywych, estymacja wektora stanu, laserowy pomiar odległości, system dokujący, system pilotowy*

1. Introduction

PNDS utilizes the range measurements between several laser heads and ship's side to determine the ship's horizontal plane contour presented on the electronic chart screen. Basing on known sensors' positions in the assigned reference system the parameters of measured points are calculated. Then the ship's position and heading can be found via such translation and rotation that result in the best fit of the measured points to the model ship outline contour. To achieve this, the common reference system for measured points and model points must be adopted and the model ship contour built from several significant points must be approximated by continuous spline functions.

2. Approximation of the model ship contour

In general, the i^{th} spline function for a cubic spline can be written as [1], [2]:

$$s_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3 \quad (1)$$

For n data points, there are n–1 intervals and thus 4(n–1) unknowns to evaluate to solve all the spline function coefficients a_i, b_i, c_i, d_i. One condition requires that the spline function goes through the first and last point of the interval, yielding 2(n-1) equations of the form:

$$\begin{aligned} & s_i(x_i) = f_i \Rightarrow a_i = f_i \\ & s_i(x_{i+1}) = f_i \Rightarrow s_i(x_{i+1}) = a_i + b_i(x_{i+1} - x_i) + c_i(x_{i+1} - x_i)^2 + d_i(x_{i+1} - x_i)^3 = f_i \end{aligned} \quad (2)$$

Another condition requires that the first derivative is continuous at each interior point, yielding n–2 equations of the form:

$$s_i^{'}(x_{i+1}) = s_{i+1}^{'}(x_{i+1}) \Rightarrow b_i + 2c_i(x_{i+1} - x_i) + 3d_i(x_{i+1} - x_i)^2 = b_{i+1} \quad (3)$$

A third condition requires that the second derivative is continuous at each interior point, yielding n–2 equations of the form:

$$s_i^{''}(x_{i+1}) = s_{i+1}^{''}(x_{i+1}) \Rightarrow 2c_i + 6d_i(x_{i+1} - x_i) = 2c_{i+1} \quad (4)$$

These give 4n–6 total equations. Two additional equations are derived assuming clamped end conditions – the first derivatives at the first and last knots are known or in other words the slope of the function at the first and last knots is set according to expertly presumed extrapolation or according to the cubic Hermite interpolant (practically also the requirement in the third condition of second derivative continuity can be changed to setting the slopes in such a way that $s_i(x)$ preserves the shape of the data and respects monotonicity).

Fig. 1 and table 1 present the horizontal plane contour of m/s "Nawigator XXI" (training-research vessel of Maritime University of Szczecin) in the adopted metric Cartesian reference system centred along x-axis at ship's conning position in relation to assigned laser heads positions while at berth. The ship's model horizontal plane contour of "Nawigator XXI" utilized in PNDS research consists of 16 significant points determined in 2D-frame (marked by green diamonds in the fig. 1). Laser heads positions translated to berth edge are marked by red triangles.

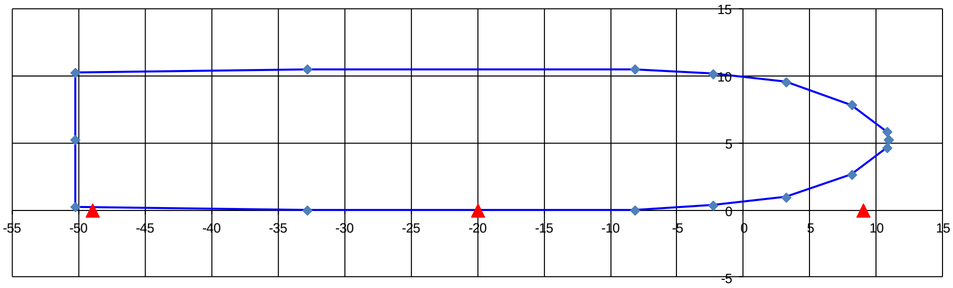

Fig. 1 The model horizontal plane contour of m/s "Nawigator XXI"

All coordinates in the table 1 are forming the array (matrix) *XY_ship*. Table 2 presents laser heads positions forming the array *XY_sensor*.

Table 1 The horizontal plane contour of m/s "Nawigator XXI" built from 16 significant points in the assigned reference system

No.	*x*	*f*(*x*)
1	10.91	5.26
2	10.8	4.66
3	8.15	2.66
4	3.2	0.96
5	-2.3	0.36
6	-8.18	0
7	-32.84	0
8	-50.3	0.26
9	-50.31	5.26
10	-50.3	10.26
11	-32.84	10.52
12	-8.18	10.52
13	-2.3	10.16
14	3.2	9.56
15	8.15	7.86
16	10.8	5.86

Table 2 Sensors positions in the assigned reference system

No.	*x*	*y*
1	-49	0
2	-20	0
3	9	0

By using spline approximation the 4-th order piecewise polynomials representing the starboard half of ship's outline have been obtained in MATLAB software. Table 3 contains the *k*=4 coefficients of the *i*-th (*i*=8) polynomial piece sorted from aft to fore. A *k*-th order piecewise polynomial has the form (by generalization of the equation (1)):

$$s_i(x) = c_{i,k}x^{k-1} + c_{i,k-1}x^{k-2} + \ldots + c_{i,2}x + c_{i,1} \tag{5}$$

It has *k* coefficients, some of which can be 0, and maximum exponent *k*-1. The order of a polynomial is one greater than its degree so a cubic polynomial is of order 4.

Table 3 Coefficients of piecewise polynomials (5) representing the half of m/s "Nawigator XXI" horizontal plane outline

i \ *k*	4	3	2	1
1	4996692	-49938.3	-500.286	5.26
2	0.000049	0.002555	-0.04465	0.26
3	0	0	0	0
4	-0.00127	0.017856	0	0
5	0.000912	0.000512	0.078677	0.36
6	-0.00119	0.041516	0.167103	0.96
7	0.100834	-0.16754	0.490593	2.66
8	-292.586	66.07165	1.726949	4.66

The final equation of the function describing the half of m/s "Nawigator XXI" horizontal plane contour takes form:

$$s(x) = \begin{cases} c_{1,4}x^3 + c_{1,3}x^2 + c_{1,2}x + c_{1,1}, & -50.31 \le x < -50.3 \\ c_{2,4}x^3 + c_{2,3}x^2 + c_{2,2}x + c_{2,1}, & -50.3 \le x < -32.84 \\ c_{3,4}x^3 + c_{3,3}x^2 + c_{3,2}x + c_{3,1}, & -32.84 \le x < -8.18 \\ c_{4,4}x^3 + c_{4,3}x^2 + c_{4,2}x + c_{4,1}, & -8.18 \le x < -2.3 \\ c_{5,4}x^3 + c_{5,3}x^2 + c_{5,2}x + c_{5,1}, & -2.3 \le x < 3.2 \\ c_{6,4}x^3 + c_{6,3}x^2 + c_{6,2}x + c_{6,1}, & 3.2 \le x < 8.15 \\ c_{7,4}x^3 + c_{7,3}x^2 + c_{7,2}x + c_{7,1}, & 8.15 \le x < 10.8 \\ c_{8,4}x^3 + c_{8,3}x^2 + c_{8,2}x + c_{8,1}, & 10.8 \le x \le 10.91 \end{cases} \tag{6}$$

where $c_{i,k}$, *i*=1,2,...,8, *k*=1,2,...,4 are given in the table 3.

The resultant approximation by 100 points is presented in the fig. 2 as the red dashed line.

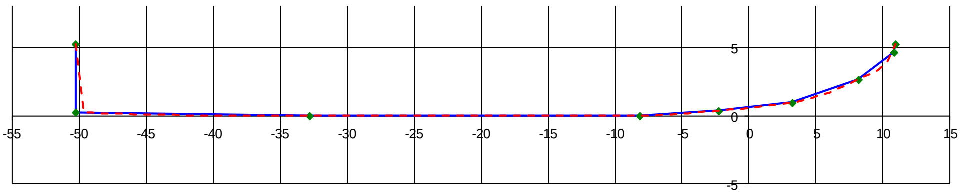

Fig. 2 The spline approximation of m/s "Nawigator XXI" half plane contour

3. Finding ship's position and heading from laser ranges measurements

The three sensors measure distances y_1, y_2, y_3 to ship's board with preset frequency. Basing on known sensors' positions, in the adopted reference system as in the fig. 1, the parameters of measured points are calculated. The next step is finding the ship's position and heading via such rotation and translation of the measured points that result in best fit to model ship horizontal plane contour. The determination of the sides which are seen by the laser sensors is necessary because of the ill-conditioned first spline piece equation (the stern is practically perpendicular to board making the function approximation indefinite). So before applying rotation the measurements can represent either L- or an I-shape. This depends on whether all three sensors are tracking a board or two of them board and one a stern (see fig. 3).

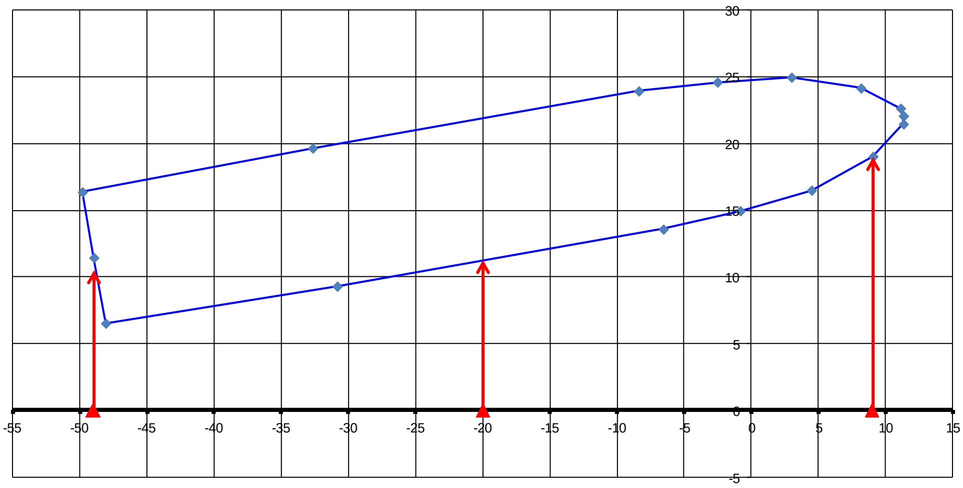

Fig. 3 Principle of measurement from three laser range sensors in PNDS

The algorithm's assumptions comprise:

– starboard side berthing (horizontal plane determination while port side berthing can be done by analogy via symmetry),
– fixed quantization parameters of ship's horizontal plane position and heading approximation in Cartesian coordinate system: 1cm×1cm×0.1°.

Then the following procedure is executed.
Firstly, the check is done if 1) all three sensors or 2) two or 3) only one or 4) none of the sensors perform measurements.
In case of 4) the algorithm stops with a warning of "no tracking – vessel outside sensors range".
In case of 3) the measured point coordinates are calculated and graphically presented and three types of warnings are given: a) "underdetermined tracking by one sensor - vessel too far aft" if sensor no. 1 performs measurements, b) "underdetermined tracking by one sensor - vessel too far forward" if sensor no. 3 performs measurements and c) "underdetermined tracking by one mid sensor – angle of approach too high".
In case of 2) the two measured points coordinates are calculated and graphically presented and two types of warnings are given: a) "underdetermined tracking by two sensors - vessel too far aft" if tracked by the sensors 1 & 2, b) "underdetermined tracking by two sensors - vessel too far forward" if tracked by the sensors 2 & 3.
In case of 1) to get the best fit to the model 2D half plane contour (6) the measurements from three sensors (x_m, y_m), m=1,2,3, have to be translated and rotated.
Without assumption that half plane contour approximated by (6) have distinct spline function values other the whole domain, the algorithm will follow analogically but for a different model ship horizontal plane contour *XY_ship* and formula (6). For "Nawigator XXI" the contour rotated 10° to portside around (x_2,0) brings satisfactory results.

Secondly, the translation of the measurement points by vector $v_j=[j\times 0.01,-y_2]$, where j=-200,-199,...,200, takes place:

$$[x\_t_{j,m}, y\_t_{j,m}] = [x_m, y_m] + v_j \tag{7}$$

The values of j are arbitrary limited and set for a given ship with resolution corresponding to the assumed accuracy of 1cm – the "Nawigator XXI" can be tracked by three sensors if the translation from model position forward and aft does not exceed 2m. The y_2 measurement is used in translation because of further rotations around (x_2,0) which is the common point of berth and model ship's plane.
After translation each set of the translated m points is further transformed via rotation around (x_2,0). The rotation transformation is given by combination of two translations and one rotation:

$$[x\_tr_{l,j,m}, y\_tr_{l,j,m}] = \left(R(\psi_l)\cdot([x\_t_{j,m}, y\_t_{j,m}] - [x_2,0])^T\right)^T + [x_2,0] \tag{8}$$

where $R(\psi_l)$ is the rotation matrix and ^{T} transposition operator:

$$R(\psi_l) = R\left(l\times 0.1^\circ\right) = \begin{bmatrix} \cos\left(l\times 0.1^\circ\right) & \sin\left(l\times 0.1^\circ\right) \\ -\sin\left(l\times 0.1^\circ\right) & \cos\left(l\times 0.1^\circ\right) \end{bmatrix} \tag{9}$$

The values of *l* (positive to starboard side) are arbitrary limited and set for a given ship with resolution corresponding to the assumed accuracy of 0.1° and depending on a), b), c) possibilities. In case of a) the "Nawigator XXI" can be tracked by three sensors if the rotation from model position does not exceed ±10° so *l*=-100,-99,...,100. In case of b) the rotation can only be negative (to the port side) so *l*=-300,-299,...,0 and the "Nawigator XXI" can be tracked by three sensors if the rotation from model position does not exceed -30°. In case of c) the rotation can only be positive (to the starboard side) so *l*=0,1,...,300.

Finally, the formula (10) is evaluated (root-mean-square deviation (RMSD) or root-mean-square error (RMSE)) in order to find the best curve fitting variant:

$$g_{l,j,m} = \sqrt{\frac{\sum_{m=1}^{3}\left(s(x\_tr_{l,j,m}) - y\_tr_{l,j,m}\right)^2}{m}} \qquad (10)$$

In the formula (10) the spline function $s(x\_tr_{l,j,m})$ is extended outside the domain set in (6) with numerically infinite values.

The goal function is to achieve: $\min(g_{l,j,m})$ or to reach the $g_{l,j,m}$ meeting the accuracy tolerance criterion, for instance:

$$g_{l,j,m} \le 0.05 \qquad (11)$$

if the total error should not differ from model more than 5cm.

The last step is to calculate coordinates of ship's horizontal plane, corresponding to criterion (11), by reverse translations and rotations:

$$[x\_f_i, y\_f_i] = \left(R(-\psi_l) \cdot \left([x_i, s(x_i)] - [x_2, 0]\right)^T\right)^T + [x_2, 0] - v_j \qquad (12)$$

Block diagram of the algorithm is presented in the following fig. 4.

In order to make the algorithm more robust against measurement outliers, it can be further extended with constraints describing the geometrical properties of the vessel hull similarly as in [3]. For instance the fact that ships have a convex outline contour can be used. If described in the y-x coordinate system (or *s*(*x*)-x) the convexity of the vessel becomes mathematically a concave constraint defined by (13).

$$s\left(\frac{x_0 + x_1}{2}\right) \ge \frac{s(x_0) + s(x_1)}{2} \qquad (13)$$

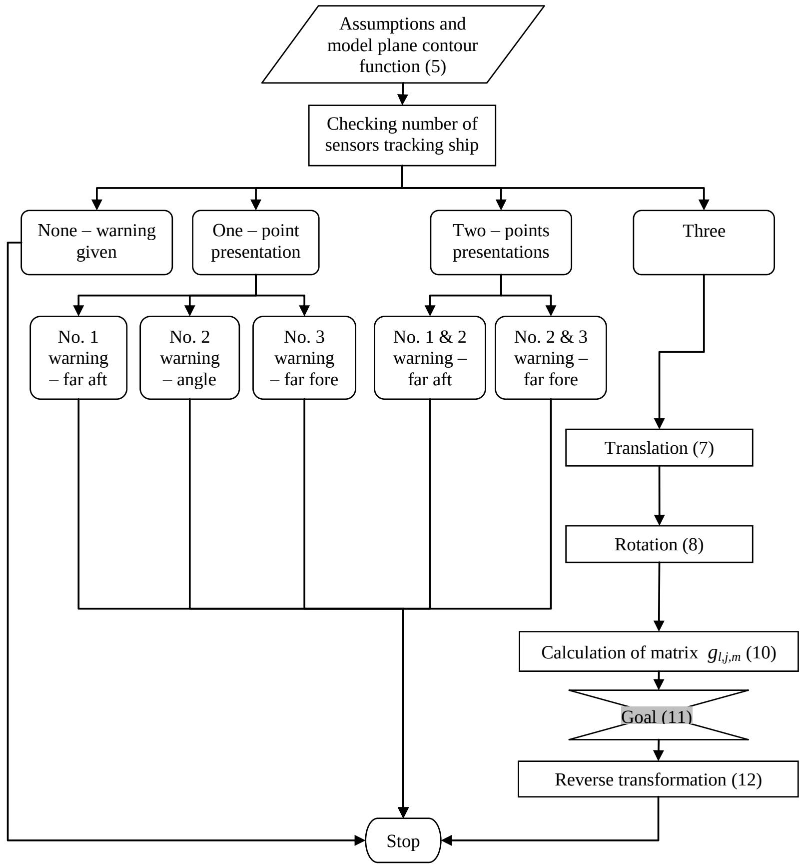

Fig. 4 Algorithm of ship's horizontal plane determination from three laser range sensors in PNDS

4. Conclusions

Modern docking assisting systems (PNDSs) utilize the range measurements between several laser heads and ship's board with preset frequency. Basing on known sensors' positions, in the adopted reference system, the parameters of measured points are calculated. If at least three sensors are used the ship's position and heading can be found via such rotation and translation of the measured points that result in best fit to model ship horizontal plane contour approximated by cubic spline function. Still, the quantization parameters and goal function's criterion have a big impact on calculation time and total algorithm's accuracy. In algorithm tests, using MATLAB implementation on PC with Core™i7 processor, it took about 3 seconds from measurements upload to final results. Implementing the code directly into C++ or C# speeds the calculation up to reasonable refresh rate of 2s, but higher demands on quantization (especially of rotation angle) slow it down dramatically.

Fig. 5 (scaled in metres) shows results of ship's horizontal plane contour determination by measurements of: 18.02m, 13.00m, 11.69m (marked by crosses). Detected angle of rotation in reference to quay line (horizontal "0" axis) was +9.4°. The goal function achieved 0.02m. The zoom of the second measurement shows similar approximation error.

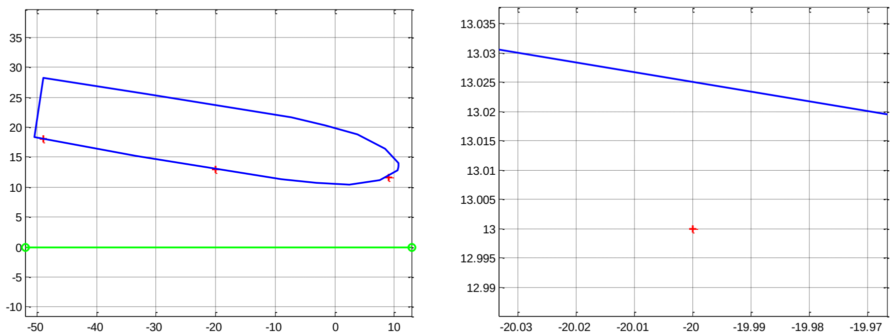

Fig. 5 Results of implementation of ship's contour fitting algorithm in MATLAB

The complete accuracy analysis taking into consideration mathematical model of ship's curve fitting and real measurements errors will be performed in the next stage of research.

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